



Significance Study on the Linear Algebra and Matrix in Mathematics

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ABSTRACT :- A significance study on the linear algebra and matrix in mathematics. Linear algebra is the branch of mathematics concerned with the study of vectors, linear spaces, linear transformation and system of linear equations. Linear transformation is a special class of function $b = f(x)$ where the independent variable x is vector in R^n and the dependent variable b is vector in R^m . Linear algebra and Matrix have many important applications in physics, engineering, social sciences, analytic geometry.

KEYWORDS :- Linear map, linear transformation, series circuit, parallel circuit. autosomal inheritance, dominant and recessive, x linked inheritance

I. INTRODUCTION

Linear algebra is widely used in both abstract algebra and functional analysis. Linear algebra has a concrete representation in analytic geometry and it generalized in operator theory [2]. It has extensive applications in the natural sciences, social sciences. Constructing curves and surfaces through specific points is an interesting application of linear algebra. Transition matrix of the population is used to find genotype distribution of the future generations [1].

II. RELETED WORK

2.1) A linear transformation $T : R^n \rightarrow R^m$ is defined by equation of the form

$$a_{11}X_1 + a_{12}X_2 + \dots + a_{1n}X_n = b_1$$

$$a_{21}X_1 + a_{22}X_2 + \dots + a_{2n}X_n = b_2$$

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ \dots \\ X_n \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ \dots \\ b_n \end{pmatrix}$$

$a_{m1}X_1 + a_{m2}X_2 + \dots + a_{mn}X_n = b_m$
In the matrix form

Or $AX=B$

The Matrix $A = [a_{ij}]$ is called the standard matrix for the linear transformation T , and T is called multiplication by A . The linear transformation $T : R^n \rightarrow R^m$ is denoted by $Ta : R^n \rightarrow R^m$ and thus $TA(X)=AX$, The vector X in R^n Is expressed as column matrix.

2.2) Let $P_1(x_1, y_1, z_1)$, and $P_2(x_2, y_2, z_2)$ be any two points in 3-space then the distance d between them is the norm of the vector

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$



- 2.3) Equation of a line : (General form) $ax + by + c = 0$
- Equation of a circle : (General form) $ax^2 + by^2 + cx + dy + e = 0$
- Equation of a conic section: (General form) $ax^2 + by^2 + cxy + dx + ey + f = 0$

III. METHODOLOGY

3.1) Constructing curve and surfaces passing through specified points.

Constructing curves and surfaces through specified points is an interesting application of linear algebra which we will discuss it now. We have a brief introduction and then technical terms and procedures from Linear Algebra that is used in the application [4]. Then few examples offered representing type of the application problems which is followed by some exercises to extend your knowledge and sharpen your skills.

Problem 1 :- Find the equation of the circle passing through the three points $A = (1,4)$, $B = (-2,3)$, $C = (7,1)$.

Solution :- General form of an equation of a circle can be written as

$$ax^2 + by^2 + cx + dy + e = 0.$$

Substitute the general point (x, y) and the three given points in the equation to form the following homogenous

$$\begin{cases} (x^2 + y^2)a + xb + yc + d = 0 \\ (1 + 16)a + (1)b + 4c + d = 0 \\ (4 + 9)a + (-2)b + 3c + d = 0 \\ (0 + 1)a + (0)b + (-1)c + d = 0 \end{cases}$$

The linear system in the matrix form can be written as $AX = 0$ with the coefficient matrix A

$$A = \begin{bmatrix} x^2 + y^2 & x & y & 1 \\ 17 & 1 & 4 & 1 \\ 13 & -2 & 3 & 1 \\ 1 & 0 & -1 & 1 \end{bmatrix}, \quad X = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\det(A) = 0.$$

This linear system has a non-trivial solution if and only if

So, let

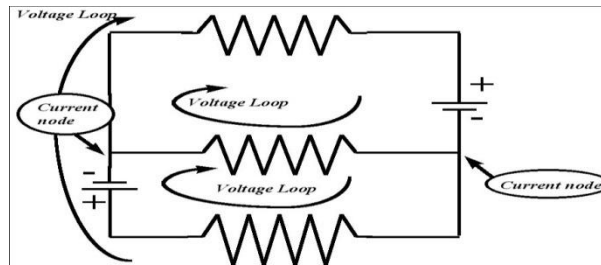
$$\begin{vmatrix} x^2 + y^2 & x & y & 1 \\ 17 & 1 & 4 & 1 \\ 13 & -2 & 3 & 1 \\ 1 & 0 & -1 & 1 \end{vmatrix} = 0$$

Use cofactor expansion along the first row, to obtain

$$(x^2 + y^2) \begin{vmatrix} 1 & 4 & 1 \\ -2 & 3 & 1 \\ 0 & -1 & 1 \end{vmatrix} - x \begin{vmatrix} 17 & 4 & 1 \\ 13 & 3 & 1 \\ 1 & -1 & 1 \end{vmatrix} + y \begin{vmatrix} 17 & 1 & 1 \\ 13 & -2 & 1 \\ 1 & 0 & 1 \end{vmatrix} - \begin{vmatrix} 17 & 1 & 4 \\ 13 & -2 & 3 \\ 1 & 0 & -1 \end{vmatrix} = 0$$

Simplify to get the equation of the circle $14(x^2 + y^2) - 4x - 44y - 58$. The above method described in problem 1 and 2 can be used to find equation of a conic section (a parabola, hyperbola or ellipse), the general form of these section is given by $ax^2 + by^2 + cx + dy + e = 0$

3.2) **Electric circuits** :- A simple **Electric Circuit** is a closed connection of Batteries , Resistors , Wires. An Electric circuit consist of voltage loops and current nodes. [4-6]



The following physical quantities are measured in an electrical circuit;

Current : Denoted by **I** measured in Amperes (A).

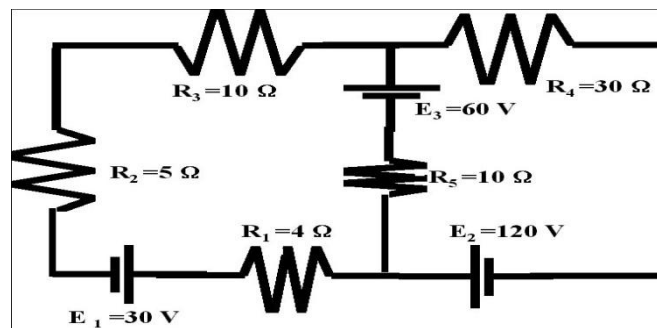
Resistance : Denoted by **R** measured in Ohms (Ω).

Electrical Potential Difference : Denoted by **V** measured in volts. (v)

Useful

Rules

We can apply the methods for solving linear systems to solve problems involving electrical circuits. In a given circuit if enough values of currents, resistance, and potential difference is known, we should be able to find the other unknown values of these quantities. We mainly use the Ohm's Law , Kirchoff's Voltage Law and Kirchoff's current Law.



Example: Find the currents in the circuit for the following network.

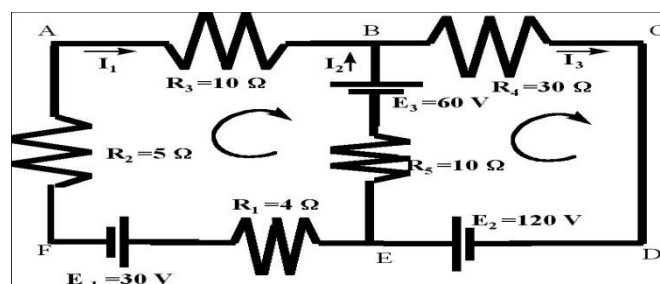
Solution : Lets assign currents to each part of the circuit between the node points. We have two node points Which will give us three different currents. Lets assume that the currents are in clockwise direction . So the current on the segment EFAB is I_1 , on the segment BCDE is I_3 and on the segment EB is I_2

Using the Kirchoff's current Law for the node B yields the equation

$$I_1 + I_2 = I_3.$$

For the node E we will get the same equation. Then we use Kirchoff's voltage law

$$-4I_1 + (-30) - 5I_1 - 10I_1 - 60 + 10I_2 = 0$$



When through the battery from (-) to (+), on the segment EF, potential difference is -30, and on segment FA moving through the resistor of 5Ω will result in the potential difference of $-5 I_1$ and in a similar way we can find the potential differences on the other segment of the loop EFAB.

In the loop BCDE, Kirchoff's voltage law will yield the following equation:

$$-30 I_3 + 120 - 10 I_2 + 60 = 0$$

Now we have three equations with three unknowns:

$$I_1 + I_2 - I_3 = 0$$

$$-19 I_1 + 10 I_2 = 90$$

$$-10 I_2 - 30 I_3 = -180$$

This linear system can be solved by methods of linear Algebra. Linear Algebra is more useful when the network is very complicated and the number of the unknowns is large.

The system above has the following solution: $I_1 = -1.698$ $I_2 = 5.7736$ $I_3 = 4.0755$

IV. RESULTS AND DISCUSSION

4.1) Genetics :- Markov chain is a mathematical model which depends on the probabilities, and is very useful in many science field, especially in genetics. Genetics is the study of genes and is part of biology. Some problems in genetics can be solved using the Markov chain [7]. For example if we know the distribution of the present generation, call it, X_0 we can use the transition matrix A of the population to find genotype distribution X_n of the future generations.

Let X_0 be initial genotype distribution of a population. Assume that the transition matrix for the genotype of given population is known, call it A . One year later, the genotype of the population, X_1 , is equal the product of the transition matrix A and the initial matrix X_0 .

$$X_1 = AX_0. \tag{4.1,1}$$

Two years later, the genotype of the population can be compute similarly,

$$X_2 = AX_1 = A(AX_0) = A^2 X_0. \tag{4.1,2}$$

And n years later,

$$X_n = A^n X_0. \tag{4.1,3}$$

According to **autosomal inheritance**, each inherited trait is assumed to be governed by a set of two genes. We designate these two genes by A and a . Each individual in the population under consideration has two of these genes, as AA , aa or Aa

Consider eye coloration, in humans. Genotype AA and Aa means having brown eyes and genotype aa means having blue eyes. Gene A in this case is called **dominant** (or we say A dominates a) and gene a is called **recessive** (or we say a is recessive to A).

[How genes of parents are passed onto their offspring?](#)

4.2) Example 1:- A common problem in this field is finding the probability of certain genotype after a given number of years. For example, suppose we want to study the fractions of the three genotypes in the n^{th} generation of cows in terms of the initial genotype fractions. (See the following example.)

Professor Vetar, at UC Davis, discovers that cows with genotype AA can produce a better quality milk than other genotypes. Professor Vetar is interested in discovering the fraction of offspring cows with genotype AA . If professor Vetar choses to cross only genotype AA with other genotypes, what are the probabilities of the offspring being AA , Aa , or aa ?

To analyze the problem, we will consider three cases:

First assume, the crossing of AA with AA . This will always gives the genotype AA , therefore the probabilities of an offspring to be AA , Aa , and aa respectively are equal to 1, 0, and 0.



Second, assume crossing of Aa with AA . The offspring will have half chance to be of genotype AA and half chance the genotype Aa , therefore the probabilities of AA , Aa , and aa respectively are $1/2$, $1/2$, and 0 .

Third, consider crossing of aa with AA . This will always results in genotype Aa . Therefore, the probabilities of genotypes AA , Aa , and aa respectively are 0 , 1 , and 0 , respectively.

The following matrix is the result of the pervious observation:

$$A = \begin{bmatrix} 1 & 1/2 & 0 \\ 0 & 1/2 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

This matrix can be viewed as the following table:

| parents genotype | | | Genotype of offspring |
|------------------|---------|-----------|-----------------------|
| $AA - AA$ | AA_Aa | $AA - aa$ | |
| 1 | $1/2$ | 0 | AA |
| 0 | $1/2$ | 1 | Aa |
| 0 | 0 | 0 | aa |

Assume that the initial population of cows made up of an equal number of each genotype, therefore, the initial

distribution vector x_0 is given by :

$$x_0 = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$$

One year later, the distribution is

$$x_1 = Ax_0 = \begin{bmatrix} \frac{1}{3}(1 + \frac{1}{2}) \\ \frac{1}{3}(\frac{1}{2} + 1) \\ 0 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1/2 \\ 0 \end{bmatrix}$$

After another year passed by, the distribution vector can be obtained as follow:

$$x_2 = Ax_1 = A(Ax_0) = A^2x_0 = \begin{bmatrix} 3/4 \\ 1/4 \\ 0 \end{bmatrix}.$$

For any positive integer n , that is n years later,

$$x_n = A^n x_0 = \begin{bmatrix} 1 & \sum_{k=1}^n \frac{1}{2^k} & \sum_{k=1}^{n-1} \frac{1}{2^k} \\ 0 & \frac{1}{2^n} & \frac{1}{2^{n-1}} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 - \frac{1}{2^{n+1}} & 1 - \frac{1}{2^n} \\ 0 & \frac{1}{2^n} & \frac{1}{2^{n-1}} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix}$$



Now if n gets larger and larger, the matrix A^n approaches to

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$x_n = A^n x_0$$

Therefore will approach to

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Suppose someone is interested to see the number of cows with genotype AA, Aa, and aa after 20 generations. One way

$$A^{20} x_0$$

to answer this question is to compute which may result in computational error during the matrix multiplication. Another approach using of diagonalization reduces computation. If the matrix A can be written as product of an invertible matrix P a diagonal matrix D and inverse of P , that is $A = PDP^{-1}$, then computation will be much simple and therefore much less computational error. This is mainly because of the following fact:

$$A^n = PD^nP^{-1} \quad \text{for } n = 1, 2, \dots$$

and

$$D^n = \begin{bmatrix} \lambda_1 & 0 & 0 & \dots & \dots & 0 \\ 0 & \lambda_2 & 0 & \dots & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & \dots & \lambda_k \end{bmatrix}^n = \begin{bmatrix} \lambda_1^n & 0 & 0 & \dots & \dots & 0 \\ 0 & \lambda_2^n & 0 & \dots & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & \dots & \lambda_k^n \end{bmatrix}$$

Not every matrix A is diagonalizable. For A to be diagonalizable it needs to have n -linearly independent eigenvectors. The matrix P is formed by writing these eigenvectors as columns of P . To find eigenvectors we will first find the eigenvalues.

For the matrix A above the eigenvalues are

$$\lambda_1 = 1, \lambda_2 = 1/2, \lambda_3 = 0$$

and corresponding eigenvectors are:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad v_2 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \quad v_3 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

The diagonal matrix

$$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

and

$$P = [v_1|v_2|v_3] = \begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

Therefore,

$$x_n = PD^nP^{-1}x_0 \longrightarrow \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

as n approach infinity..

V. CONCLUSION

We are presenting application of linear algebra and matrices in mathematics. Linear system of equation in linear algebra play an important role in geometry, electronic physics and biology. A linear equation can have one or more



variables. Linear algebra play an important role also in Biotechnology. Linear system of equation is a way to find probability of certain genotype of species after a given number of years.

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